THE POWER INPUT OF SCREW ROTORS AND AGITATORS: THE POWER INPUT OF SCREW AGITATOR WITH A DRAUGHT TUBE

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The power input has been measured of 52 various geometries of the screw agitator with a draught tube. The obtained results have been correlated by a relation derived for the power input of screw rotors in the previous paper.

Screw agitator with a draught tube is one of the most effective devices for mixing of viscous liquids. Mixing equipment with a screw agitator in a draught tube shown in Fig. 1, may be divided in two principal regions. Region *1* consists of a screw located in the draught tube and acts as a pump delivering the mixed liquid into region 2 outside the agitator proper. The region 2 may be divided into regions of flow reversal *2a* and *2c* and the annular region between the wall of the vessel and draught tube *2b*. The measurement of the power input of screw agitators has been subject of a series of works[1-4] but in only several of them[1-2] at least semiempirical equation have been proposed for the calculation of the power input. The aim of this paper is to show how to utilise the power input characteristic equation, derived in the first part of this series[5], for the calculation of the power input of screw agitators with a draught tube and to verify the proposed equation experimentally.

THEORETICAL

The mathematical model for the calculation of the pumping capacity of a screw agitator, presented in refs[6,7], starts from the facts that the pressure difference in the equation for the pumping characteristic of the screw

$$\dot{V}/nd^3 = a - b\,\Delta p/\mu n \qquad (1)$$

equals the pressure difference arising during the flow of the mixed liquid through the region 2. Since no data for the determination of the pressure drop due to the flow through the regions of flow reversal *2a* and *2c* are available their contribution to net pressure drop under the flow through the region 2 was neglected and the pressure drop was calculated for the region *2b* only from the relation presented e.g. in ref.[8]

$$\Delta p = \frac{128L/d}{\pi(D/d)^4}\left\{1 - (D_t/D)^4 - \frac{[1-(D_t/D)^2]^2}{\ln(D/D_t)}\right\}^{-1}\frac{\mu\dot{V}}{d^3} = k\,\mu\dot{V}/d^3\,. \qquad (2)$$

For the calculation of the dimensionless power input from the power input characteristic equation (see Eq. (*32*) of ref.[5]) one must know the dimensionless pressure difference which shall be determined from the simultaneous solution of Eq. (*1*) and (*2*) as

$$\Delta p/(\mu n) = ak/(1 + bk)\,. \qquad (3)$$

Upon substituting Eq. (*3*) into the dimensionless power input characteristic for the dimensionless power input we obtain

$$P^* = a_p + a^2k/(1 + bk)\,. \qquad (4)$$

A number of simplifying assumptions have been made in the process of the derivation of Eq. (*4*) and the starting equations. Accordingly, the accuracy of Eq. (*4*) had to be tested experimentally. Our own measurements have been used for this test on the one hand and the results of power input measurements published by Seichter[1] on the other hand.

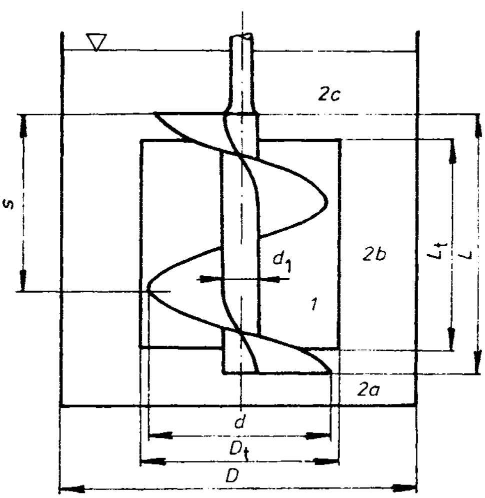

FIG. 1
Screw agitator with a draught tube

EXPERIMENTAL

The experimental verification of Eq. (*4*) was carried out by measuring the power input on 52 geometries of the screw agitator with a draught tube. These geometries were combination of 19 screw rotors (see Table I) and 22 draught tubes (see Table II).

The power input measurements were carried out on an adapted viscometer RHEOTEST. Water solutions of starch syrup were used as model liquids. Average values of P^* obtained in the creeping flow region for individual configurations are summarized in Table III.

RESULTS

From Table III it follows that the agreement between the experimental data and the values computed from Eq. (*4*) is good. The average difference amounts to about 0·2%. The maximum deviation for precisely lathed screw agitators is 11%, for less accurate welded agitators 14% (agitators I–VI).

A comparison of the values computed from Eq. (*4*) with those measured by Seichter[1] is given in Table IV. The table shows the average deviation of the experimental and computed data to be 2·5%. The maximum difference is 8% which is thought

TABLE I

The geometry of the screw agitator; the geometry of rotors

Number of agitator	d, mm	d_1, mm	s, mm	L, mm	e, mm
I	50·8	17	50	75	1
II	60·0	14	60	90	1
III	94·0	17	94	141	1
IV	50·0	10	50	146	1
V	60·3	12	60	139	1
VI	74·75	15	75	140	1
VII	100	20	100	188	1
VIII	75	15	75	112·5	2
IX	75	30	75	112·5	2
X	75	45	75	112·5	2
XI	94·2	18·8	94	141	2
XII	94·3	37·6	94	141	2
XIII	94·4	56·4	94	141	2
XIV	75	15	112·5	112·5	2·2
XV	75	30	112·5	112·5	2·2
XVI	75	45	112·5	112·5	2·2
XVII	75	15	150	105	2·4
XVIII	75	15	75	75	2·4
XIX	94·0	18·8	94	94	2·0

to be good considering that the agitators were welded. One may thus conclude that the proposed equation for the calculations of the power input of the screw agitators with a draught tube describes well the experimental data which not only confirms the utility of Eq. (*4*) but also the more general power input characteristic equation of screw rotors (Eq. (*32*) in ref.[5]).

Eq. (*4*) may be used not only for the calculation of the power input of screw agitators but also for the determination of the effect of main geometrical parameters on the power input. This is shown graphically (solid line and points) in Figs 2–5 plotting also the given dependences (broken line) computed from the following equation recommended by Seichter[1,9].

$$P^* = P_D^* + 25 \cdot 9 \left(\frac{D}{d}\right)^{-4.59} \left(\frac{s}{d}\right)^{1.71} \left(\frac{d_1}{d}\right)^{-1.7} \frac{L}{d}, \qquad (5)$$

where for the dimensionless power input under the drag flow the following relationship is recommended

$$P_D^* = 2\pi^3 \frac{L_t}{d} \frac{D_t}{d} \left[\frac{d}{D_t - d_1} \frac{s - e}{s} F_z(\cos^2 \varphi_t + 4 \sin^2 \varphi_t) + \frac{d}{D_t - d} \frac{e}{s}\right]. \qquad (5a)$$

F_z is a correction coefficient recommended, after respecting the effect of the flight by Strub[10]. Further the Figs 2–5 show also (dash and dot line) a comparison with

TABLE II

The geometry of the screw agitator; the geometry of draught tubes

Number of draught tube	D_t, mm	L_t, mm	D, mm	Number of draught tube	D_t, mm	L_t, mm	D, mm
1	54·9	58	100	22	79·35	95	150
2	54·9	58	150	23	82·5	95	150
3	54·9	58	200	24	86·0	95	150
4	65·4	75·6	100	31	96·5	120	150
5	65·4	75·6	150	32	99·5	120	150
6	65·4	75·6	200	33	104	120	150
11	54·5	119	150	34	107·5	120	150
12	65·7	119	150	41	82·4	63	150
13	83·0	119	150	42	82·4	75	150
14	109·0	161	200	51	103·5	78·6	150
21	77·25	95	150	52	103·5	94	150

TABLE III

A comparison of experimental and computed values of power input

Number of geometry	P^* experiment	P^*, Eq. (4)	P^*_{calc}/P^*_{exp}
I/1	232·8 ± 3·7	242·3	1·041
I/2	208·2 ± 2·8	237·4	1·140
I/3	211·6 ± 2·7	237·0	1·120
II/4	257·3 ± 2·9	244·1	0·949
II/5	217·1 ± 4·6	223·7	1·030
II/6	211·4 ± 4·4	222·6	1·053
III/33	266·5 ± 2·8	235·5	0·884
IV/11	418·2 ± 9·0	434·4	1·039
V/12	343·0 ± 4·8	342·1	0·997
VI/13	256·0 ± 1·8	262·9	1·027
VII/14	257·4 ± 2·5	274·1	1·065
VIII/21	365·2 ± 3·5	345·2	0·945
VIII/22	273·1 ± 2·3	269·8	0·988
VIII/23	231·2 ± 2·7	229·0	0·990
VIII/24	208·5 ± 4·5	207·9	0·997
IX/21	361·7 ± 3·8	369·9	1·023
IX/22	277·2 ± 3·0	292·0	1·053
IX/23	244·0 ± 1·8	246·8	1·011
IX/24	220·6 ± 6·7	220·6	1·000
X/21	428·0 ± 5·0	429·5	1·004
X/22	340·6 ± 4·4	343·3	1·008
X/23	285·5 ± 2·8	285·5	1·000
X/24	264·9 ± 2·4	248·8	0·939
XVII/23	220·5 ± 3·0	218·8	0·992
XVIII/41	159·0 ± 2·1	156·3	0·983
XVIII/42	165·0 ± 2·7	160·0	0·970
XI/31	418·9 ± 2·9	379·5	0·906
XI/32	306·4 ± 4·5	286·8	0·936
XII/33	261·4 ± 1·7	247·4	0·946
X/34	245·6 ± 5·4	233·6	0·951
XII/31	436·5 ± 8·5	407·6	0·934
XII/32	320·2 ± 3·5	306·8	0·958
XII/33	273·0 ± 2·1	260·9	0·956
XII/34	255·8 ± 5·6	242·2	0·947
XIII/31	482·3 ± 12·7	468·6	0·972
XIII/32	350·2 ± 4·2	352·0	1·005
XIII/33	297·2 ± 4·4	289·5	0·974
XIII/34	275·4 ± 7·0	262·4	0·953
XIV/21	332·8 ± 5·8	308·4	0·927
XIV/22	262·6 ± 2·8	253·3	0·965
XIV/23	221·2 ± 1·1	221·3	1·000

TABLE III
(*Continued*)

Number of geometry	P^* experiment	P^*, Eq. (*4*)	P^*_{calc}/P^*_{exp}
XIV/24	198·0 ± 2·7	201·4	1·017
XV/21	343·9 ± 6·6	348·1	1·012
XV/22	280·5 ± 5·4	288·4	1·028
XV/23	231·8 ± 1·1	247·8	1·069
XV/24	203·7 ± 2·4	218·0	1·070
XVI/21	386·8 ± 6·9	430·3	1·112
XVI/22	324·2 ± 3·0	355·2	1·096
XVI/23	274·1 ± 2·2	290·5	1·060
XVI/24	236·2 ± 4·5	243·3	1·030
XIX/51	168·6 ± 3·8	165·7	0·983
XIX/52	180·3 ± 5·7	168·2	0·933

TABLE IV
A comparison of values computed from Eq. (*4*) with experimental data[1]

Number of configuration	D/d	d_1/d	s/d	L/d	P^*_{exp}	P^*_{calc}	P^*_{calc}/P^*_{exp}
S1	3·37	0·37	1·00	1·50	207·2	224·5	1·08
S2	2·30	0·25	0·33	1·50	473·6	469·1	0·99
S3	3·10	0·25	0·33	1·50	472·3	469·1	0·99
S4	2·30	0·25	0·46	1·50	371·2	352·0	0·95
S5	2·30	0·25	0·60	1·50	306·1	284·2	0·93
S6	2·30	0·25	0·75	1·50	244·9	246·3	1·01
S7	3·10	0·25	1·00	1·50	213·6	214·7	1·01
S8	2·30	0·25	1·33	1·50	210·3	206·9	0·98
S9	2·30	0·25	1·50	1·50	211·0	207·6	0·98
S10	3·10	0·25	1·50	1·50	217·6	204·4	0·94
S11	2·00	0·22	1·00	1·50	231·0	216·0	0·94
S12	2·69	0·22	1·00	1·50	220·8	210·8	0·95
S13	1·58	0·17	0·93	1·37	224·0	219·8	0·98
S14	2·13	0·17	0·93	1·37	204·7	194·8	0·95
S15	2·13	0·17	0·93	1·37	213·9	194·8	0·91
S16	1·97	0·22	1·00	1·50	223·8	212·4	0·95

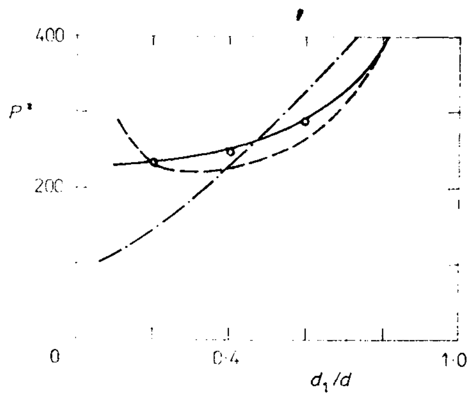

FIG. 2

Dependence of the dimensionless power input P^* on the ratio d_1/d: ● experimental values, ——— calculated from Eq. (4), – – – calculated from Eq. (5), recommended by Seichter[1], –·–·–·– computed from Eq. (6) proposed by Chavan and Ulbrecht[2]

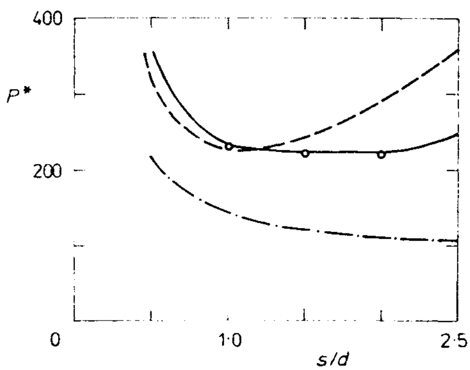

FIG. 3

Dependence of the dimensionless power input P^* on the ratio s/d (the same caption as in Fig. 2)

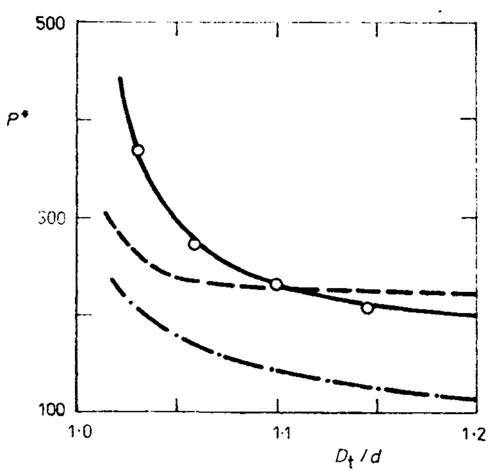

FIG. 4

Dependence of the dimensionless power input P^* on the ratio D_t/d (the same caption as in Fig. 2)

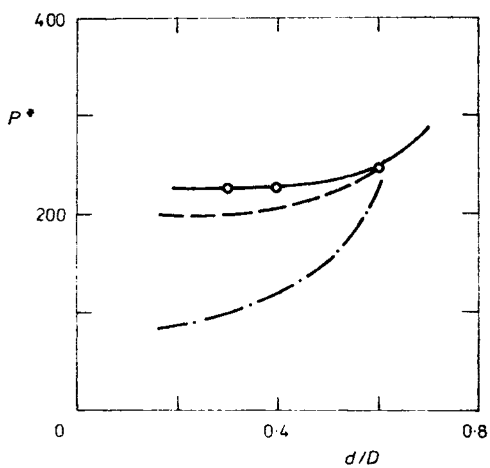

FIG. 5

Dependence of the dimensionless power input P^* on the ratio d/D (the same caption as in Fig. 2) the results computed from the following equation recommended by Chavan and Ulbrecht[2]

$$P^* = 2\pi^2 a_1 \frac{d_e}{d} \frac{\lambda^2}{\lambda^2 - 1} \left(1 + \frac{2d}{D - L}\right)^{0.37} \left(\frac{D - D_t}{L_t}\right)^{-0.46} \left(\frac{D - L_t}{2d}\right)^{-0.036}, \qquad (6)$$

where

$$\lambda = D_t/d_e \qquad (6a)$$

and for the ratio of the equivalent diameter to the agitator diameter d_e/d refs[2,11] recommend the following equation

$$\frac{d_e}{d} = \frac{D_t}{d} - 2\frac{H}{d} \Big/ \ln \frac{(D_t/d) - (1 - 2H/d)}{D_t/d - 1}. \qquad (6b)$$

For the dimensionless surface of the screw a_1 refs[2,11] recommend

$$a_1 = \frac{(L/d)(s/d)}{3\pi} \left\{\frac{\pi \sqrt{[(s/d)^2 + \pi^2]}}{(s/d)^2} + \ln\left[\frac{\pi}{(s/d)} + \frac{\sqrt{[(s/d)^2 + \pi^2]}}{(s/d)}\right].\right.$$
$$\left. . [1 - (1 - 2H/d)^2] + \pi(1 - 2H/D) L/d. \qquad (6c)\right.$$

Fig. 2 plots the dependence of the dimensionless power input P on the relative diameter of the root d_1/d. From the figure it may be seen that with increasing diameter of the root the power input grows. The solid line, computed according to Eq. (4), is in good agreement with the experimental points shown by circles. The dependence computed from Eq. (5) recommended by Seichter agrees quantitatively fairly well with experimental points. Qualitatively the agreement is also fairly good with the exception of the initial portion (for $d_1/d < 0{\cdot}2$). The dash and dot line, computed from Eq. (6) recommended by Chavan and Ulbrecht, follows the experimental points neither qualitatively nor quantitatively.

Fig. 3 shows the dependence of the dimensionless power input P^* on the relative pitch of the screw, s/d. The figure shows again the good agreement of the experimental data with the values computed from Eq. (4). From the computed curve it is apparent that the power input with increasing pitch initialy rapidly decreases and this part is followed by a flat region for $1 < s/d < 2$. For values $s/d > 2$ the power input again increases with increasing pitch. The dependence, computed from Eq. (5), recommended by Seichter does not fit the experimental points particularly in region $s/d > 1{\cdot}5$, where the computed values, unlike the real data, rapidly increase with increasing pitch. The dash and dot line, computed from Eq. (6), gives consistently lower values than the real ones. Fig. 4 shows the dependence of the dimensionless power input P^* on the ratio of the diameter of the draught tube to the diameter of the agitator. From the figure it may be seen that with increasing value of the ratio D_t/d the power input decreases and one can see again the good agreement between the experimental data and the results computed from Eq. (4). The dependence of P^* on D_t/d, computed from Eq. (5) recommended by Seichter, is much less significant than in reality, this being is apparently due to the fact that the correction coefficient F_z, computed by Strub[10] is, contrary to experiments, independent of D_t/d. The dash and dot line computed from Eq. (6) differs significantly from the experimental data and mainly qualitatively.

Finally, Fig. 5, plotting the dependence of P^* on the relative size of the agitator measured by the size of the vessel, d/D, shows that P^* depends on d/D in the investigated region insignificantly, which is at odds particularly with the dependence computed from Eq. (6).

Summarizing the inspection of the above figures one can say that recommended Eq. (4) fits well the experimental dependences, which appears particularly valuable as it is an equation derived theoretically without empirically determined correction coefficients. Eq. (5) recommended by Seichter[1] agrees qualitatively fairly well with the majority of measurements, yet, qualitatively, one can observe significant deviations particularly in the expression of the effect of the ratio D_t/d. Eq. (6) recommended by Chavan and Ulbrecht[2] yields substantially lower values of the power input, which is hazardous especially from the standpoint of undersizing the drive of the agitator. In addition this equation does not fit the experimental data well even qualitatively.

LIST OF SYMBOLS

a, b	dimensionless coefficients in Eq. (1) defined in ref.[11]
a_p	dimensionless coefficient in power characteristics (32)
a_p	dimensionless coefficient in power characteristics (32) (ref.[5])
a_1	dimensionless area of screw impeller defined in Eq. (6c)
d	diameter of agitator, m
d_1	diameter of root, m
d_c	equivalent diameter defined in Eq. (6b), m
D	diameter of vessel, m
D_t	diameter of draught tube, m
e	axial thickness of screw flight, m
F_z	correction coefficient recommended by Strub accounting for the effect of screw flight on power input
H	$= (D_t - d_1)/2$ depth of screw channel, m
L	length of screw, m
L_t	length of draught tube, m
n	frequency of revolution, m
p	pressure, Pa
Δp	pressure difference on the screw, Pa
P^*	$= P/\mu n^2 D^3$ dimensionless power input
P^*_D	dimensionless power input for drag flow according to Seichter — Eq. (5a)

s	pitch of screw, m
$\dot{V}$	pumping capacity, $m^3 s^{-1}$
λ	dimensionless ratio — see Eq. (*6a*)
μ	dynamic viscosity, Pa s
φ_t	helix angle of screw surface on diameter D_t

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